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Contraction onto the Main Sequence:

As we discussed, T_e will be constant during the collapse from $R \sim 10^5 R_\odot$ to $R \sim 50 R_\odot$ (for $M = M_\odot$). However, it will increase drastically in the subsequent phase. One reason is that the convection becomes important as temperature gradient becomes larger (note that the core temperature increases while T_e has remained constant).

A quick rise in T_e from (50 K to 4000 K) results in a rapid increase in L (by a factor of $\sim 10^6$). Larger luminosity implies that the collapsing cloud can lose energy more easily. This leads to a rather short (~300 days for a $M_\odot$ star) phase during which the gravitational potential energy is that is released due to contraction can be efficiently radiated away from the cloud.

The change in the gravitational energy over a time scale $t_{ff} \sim (G\rho)^{-\frac{1}{2}}$ is $\Delta\Omega \sim \frac{GM^2}{R}$. This results in:

$$\frac{\Delta\Omega}{t_{ff}} \propto R^{-\frac{5}{2}} \Rightarrow L \propto T_e^4 R^2 \propto R^{-\frac{5}{2}} \Rightarrow \boxed{L \propto T_e^{\frac{20}{9}}}$$

After the formation of a dense core, the luminosity is decided essentially by the nearly constant accretion rate $\dot{M}$ of the thinner outlying gas, with $L \propto \frac{GM\dot{M}}{R}$. Therefore, later on L remains constant as the outer regions become thinner and thinner.

With the steady increase of the temperature of the protostar, the opacity of the outer layers will become dominated by the H^- ion, with the extra electron provided by partial ionization of heavier elements in the gas cloud. The high opacity causes the envelope to become convective. By and large the convective layer extends all the way down to the center. We can then model this phase of evolution as a fully convective star. Such a star lies on the Hayashi line, as discussed before, in the H-R diagram. This is an almost vertical line determined by:

$$T_e \propto M^{\frac{n+3}{9n+3-2s}} \quad L^{\frac{3n-1}{9n+3-2s}} \qquad (\kappa \propto \rho^n T^{-s})$$

For H^- opacity $\kappa \propto \rho^{0.5} T^9$, which results in:

$$T_e \propto \left(\frac{M}{M_\odot}\right)^{\frac{7}{51}} \left(\frac{L}{L_\odot}\right)^{\frac{1}{102}} K \qquad *$$

For bb and bf absorptions, we have $n=1$, $s=3.5$ that leads to $T_e \propto M^{0.8} L^{0.2}$. The dependence on L is very mild in both cases, hence almost vertical line.

Contraction along the Hayashi line continues as long as the energy transport is due to convection. The evolutionary track will change when the temperature gradient favors radiative transport.

As the star moves along the Hayashi line, the internal temperature increases as $T \propto \frac{GM}{R}$. The actual temperature gradient is ;

$\nabla T \propto \frac{GM}{R^2}$

The radiative temperature gradient varies as;

$\left(\frac{dT}{dr}\right)_{rad} = \frac{-3\kappa \rho L}{16\pi a c T^3 R^2} \sim \frac{\kappa L}{\mu^3 M^2 R^2}$ (μ: mean molecular weight)

Using $\kappa \propto \rho^n T^{-s}$, we find the ratio between the two gradients as;

$\mathcal{R} \sim \frac{M^{s-n+3}}{L R^{s-3n}}$

Depending on the dominant source of opacity, the radiative gradient can exceed the actual temperature gradient as the star contracts along the Hayashi line. Once this happens, the radiative transfer will take over. The location in the H-R diagram at which this transition occurs is given by $\mathcal{R} \simeq 1$. The locus of points of constant

$\mathcal{R}$ is given by (note that $L \alpha R^2$ since T_e is essentially constant on the Hayashi line):

$$\frac{\partial \ln L}{\partial \ln M} = 2\left(\frac{s-n+3}{s-3n+2}\right)$$

Using the expression in equation *, we find:

$$\frac{\partial \ln L}{\partial \ln T_e} = \frac{2(9n+3-2s)}{n+3}\left(\frac{s-n+3}{s-3n+2}\right)$$

The intersection of this line and the Hayashi line marks the onset of a radiative core.

With the end of convection and formation of a radiative core it is the radiative opacity that determines the leakage of energy from the star. Further contraction and conversion of gravitational potential energy into radiation will proceed slowly but will lead to a steady increase in the temperature gradient and luminosity. The increased luminosity and decreased radius will make the evolutionary trajectory in the H-R diagram move to the left and to the up. This is called the Henyey track.

As the star evolves along the Henyey track, its core density increases. If this increase leads to an increase in the temperature (as happens for an ideal gas) then the core temperature will eventually reach a value at which Hydrogen burning can take place. The cloud then reaches the appropriate point on the main sequence and is stabilized by nuclear reactions.

The amount of time required for stars to collapse onto the main sequence is a decreasing function of the stellar mass. A 0.5 $M_\odot$ star takes over 10^8 yr, whereas a 15 $M_\odot$ star requires only 6×10^4 yr. Lower mass stars spend more time on the Hayashi line and less time on the Henyey track.

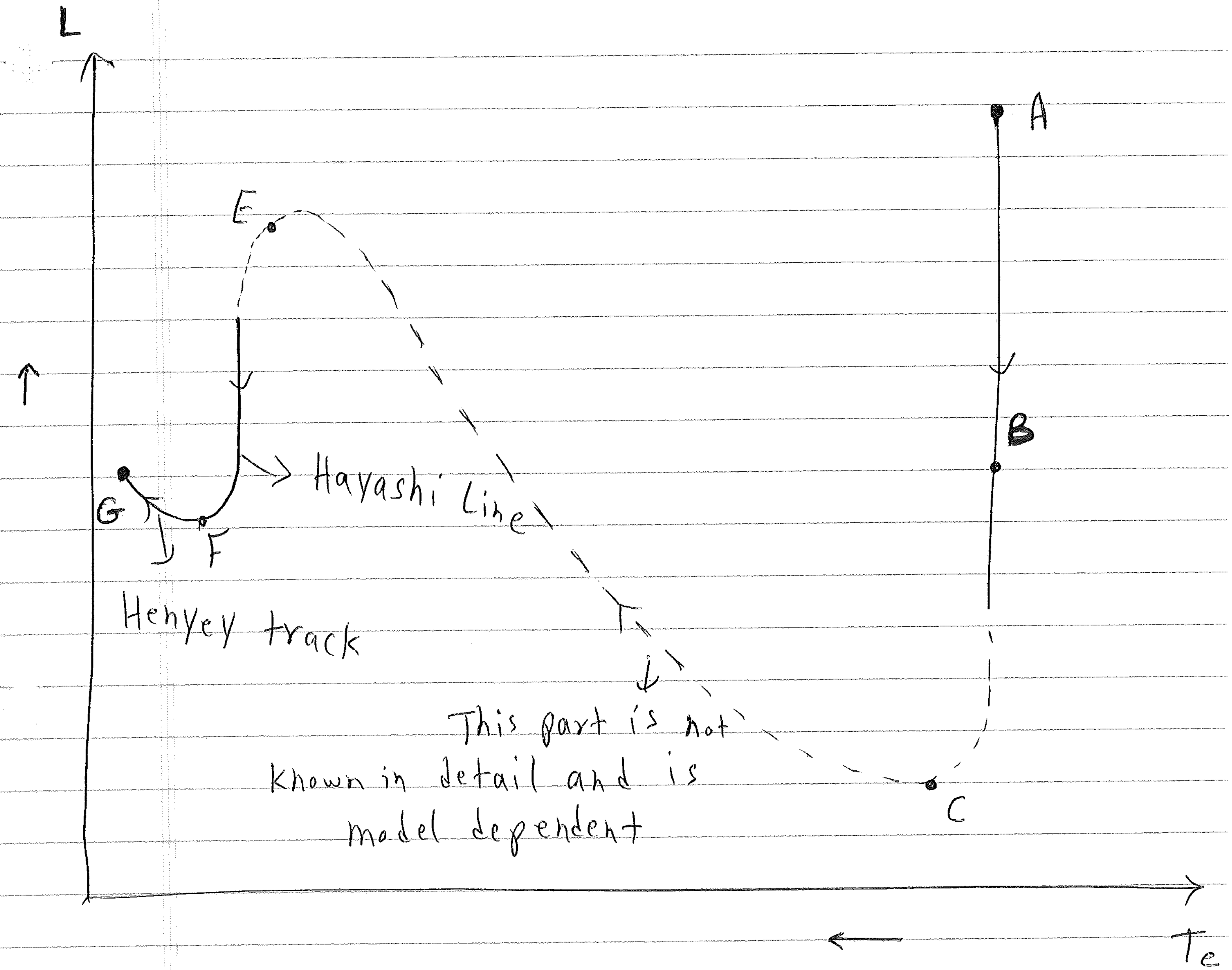
In general, lower mass cloud fragments will be more abundant inside a given interstellar cloud. Thus the number of stars that form per unit volume per unit mass interval will be a strong function of the mass. Low mass stars are formed in larger numbers and last longer, hence exhibiting greater abundance. It is not easy to obtain distribution functions for the number densities of stars of different mass ranges from the first principles. One simple parameterization that is extensively used and is based on observation is;

$$\Psi_s(m)\,dm = 2\times10^{-12}\, m^{-2.35}\, dm\ yr^{-1} pc^{-3}, \quad m \equiv \frac{M}{M_\odot}$$

This gives the birth rate of stars within the mass range $0.4\,M_\odot \lesssim M \lesssim 10\,M_\odot$.

One Comment is in order. Our discussion on star formation from the Collapse of interstellar clouds does not include effects of rotation and magnetic fields. Both of these have the effect of working against gravity and preventing the Collapse. The existence of a non-zero angular momentum for the original cloud will also cause the Collapse to be anisymmmetric instead of spherical. In extreme case, this can lead to the formation of a protostellar disk, which accretes to the central Core. There are means for the cloud to shed its angular momentum and magnetic field, but the details of this process are unclear as yet.

Finally, a schematic evolutionary trajectory of a star formed as a result of gravitational Collapse is shown in the H-R diagram:



A: Onset of Collapse

B: Core becomes opaque

C → E: Drastic rise in T_e, L

E: Motion on the Hayashi line begins

F: Switch to motion along the Henyey track

G: Main sequence point